Column Parity Mixers

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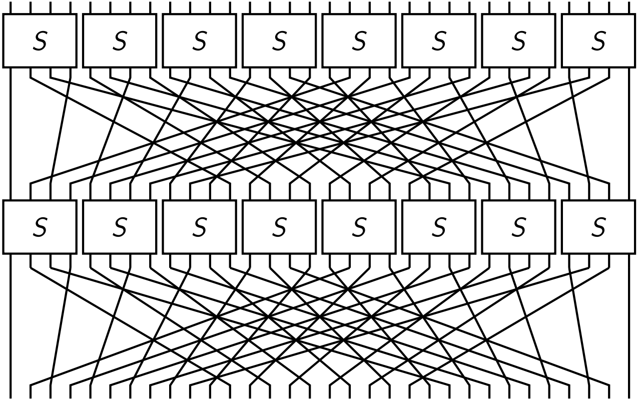
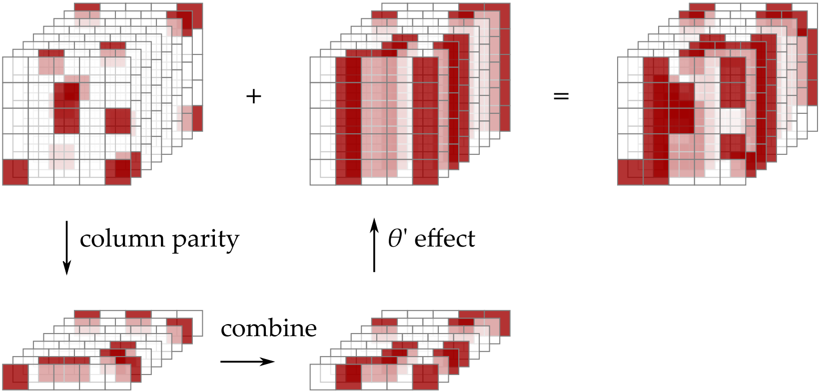
iCIS | Digital Security
Radboud University

Diffusion



Diffusion in Keccak-f



Only 2 XORs/bit + good bounds on differential trails [MDA17]

Column parity mixers

For an $m \times n$ matrix A over $\mathbb{F}_2^{\ell}$:

$$\theta(A) = A + \quad f(A)$$

$$\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}$$

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For an $m \times n$ matrix A over $\mathbb{F}_2^\ell$:

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For an $m \times n$ matrix A over $\mathbb{F}_2^\ell$:

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θ fully defined by m, n and Z

Special case: circulant Z

$$\begin{pmatrix} z_0 & z_1 & z_2 & z_3 \\ z_1 & z_2 & z_3 & z_0 \\ z_2 & z_3 & z_0 & z_1 \\ z_3 & z_0 & z_1 & z_2 \end{pmatrix}$$

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$$z(x) = z_0 + z_1 x + z_2 x^2 + z_3 x^3$$

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$$\theta(a(x,y)) = a(x,y) + \frac{1+y^m}{1+y} z(x) a(x,y) \bmod (1+x^n)(1+y^m)$$

Algebraic properties

$$\begin{aligned}\theta'(\theta(A)) &= \theta'(A + \mathbf{1}_m^m AZ) \\ &= A + \mathbf{1}_m^m AZ + \mathbf{1}_m^m AZ' + (\mathbf{1}_m^m)^2 AZZ'\end{aligned}$$

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- If m even, $(\mathbf{1}_m^m)^2 = \mathbf{0}_m^m$:
  - $\theta'(\theta(A)) = A + \mathbf{1}_m^m A(Z + Z')$
  - Group isomorphic to $\left(\mathbb{Z}_2^{n^2}, +\right)$
  - CPM is invertible, involution, commutative

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- If m odd, $(\mathbf{1}_m^m)^2 = \mathbf{1}_m^m$:
  - $\theta'(\theta(A)) = A + \mathbf{1}_m^m A\left((Z + \mathbf{I})(Z' + \mathbf{I}) + \mathbf{I}\right)$
  - Group isomorphic to $GL(n, 2)$
  - CPM is invertible iff $Z + \mathbf{I}$ is, non-commutative

Propagation properties

- Differences

  $A_\Delta = A + A'$ at the input

  $\Rightarrow B_\Delta = \theta(A) + \theta(A') = \theta(A_\Delta)$ at the output

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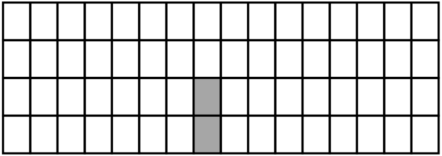
- Linear masks

  V at the output

  $\Rightarrow U = V + \mathbf{1}_m^m V Z^\mathsf{T}$ at the input

Diffusion with CPMs

- How about a state like this?



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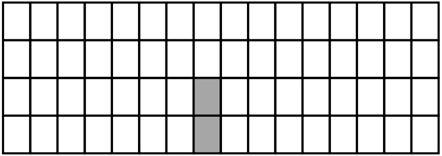

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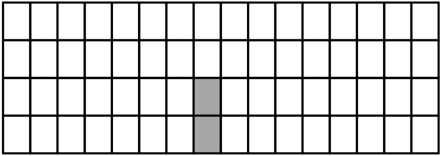

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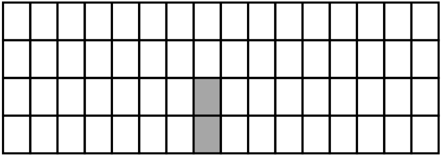

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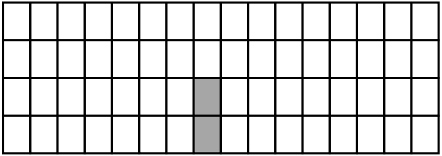

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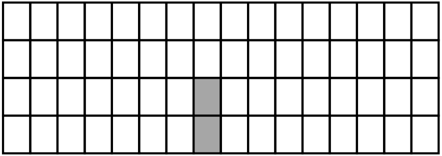

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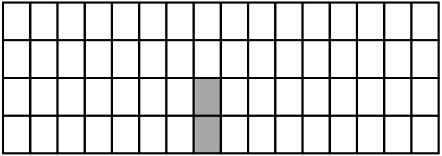

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- Single-bit difference propagates to $1 + |Z|\, m$ bits

CPMs vs. (near-)MDS matrices

Cipher	Type	XORs/bit	Branch no.
AES	MDS	3.03	5
Joltik	MDS	3	5
PHOTON	MDS	5†	7
Prøst	MDS	4.5†	5
Midori	Not MDS‡	1.5	4
Minalpher	Not MDS‡	1.5	4
Prince	Not MDS	1.5	4
SKINNY	Not MDS	0.75	2
Keccak-f	CPM	2	4
Circulant CPM	CPM	$2 + \frac{\lvert z(x)\rvert - 2}{m}$ *	4

* XORs/bit $\in [2 - 1/m, 2 + (n-2)/m]$

† Unknown whether it can be computed with less XORs

‡ Can also be considered to be a CPM!

CPM example

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

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$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$$

$$\Leftrightarrow$$

$$m = 2, Z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Building a permutation with a CPM

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12. Give it a name

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- CPM causes heavy search space branching
- Dedicated software for CPM-based ciphers/permutations

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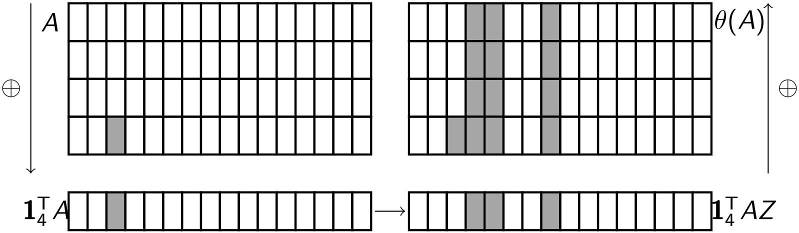

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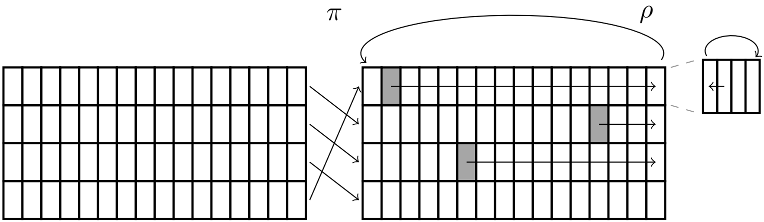

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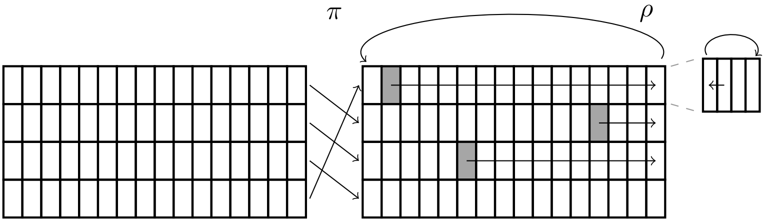

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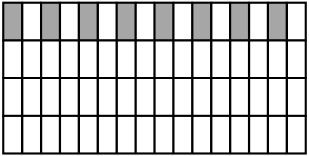
- ι: add $\texttt{0xF3485763} \gg i$ in round i to every other cell of top row



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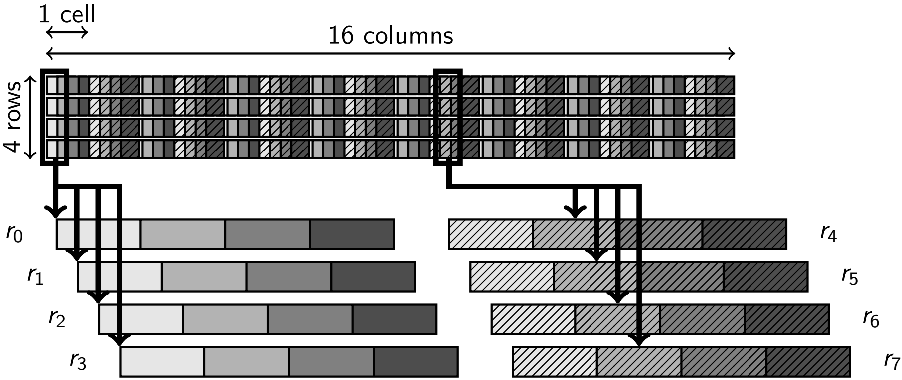
  - In kernel: ≥ 52 active cells
  - Outside kernel: ≥ 46 active cells (differential), DP 2^{-92}
  - Outside kernel: ≥ 40 active cells (linear), LP 2^{-80}
- Preliminary study makes us believe trail clustering, impossible differentials, invariant attacks are not a concern

Mixifer implementation



Mixifer comparison (ARM Cortex-M4)

Cipher	Width (bits)	r	Speed (cpb) Full	Speed (cpb) $/r$	Bound trails r	Bound trails W	Bound trails $/r$
AES bitsliced	128	10	50.52	5.05	4	150	37.5
AES tables			39.97	4.00			
Gimli	384	24	21.81	0.91	8	52	6.5
Keccak-f[400]	400	20	106	5.3	6	92	15.3
Keccak-f[800]	800	22	48.02	2.18	6	92	15.3
Salsa20/20	512	20	13.88	0.69	3	18	6
Mixifer	256	16	36.69	2.33	4	92	23

Thanks. . .

. . . for your attention

Questions?

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