Mixing Layers in Symmetric Crypto

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Part I

Shorter Linear Straight-Line Programs for MDS Matrices

Part II

Column Parity Mixers

MDS Matrices in Symmetric Crypto

- <u>M</u>aximum <u>D</u>istance <u>S</u>eparable

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- But when viewed as binary matrix:
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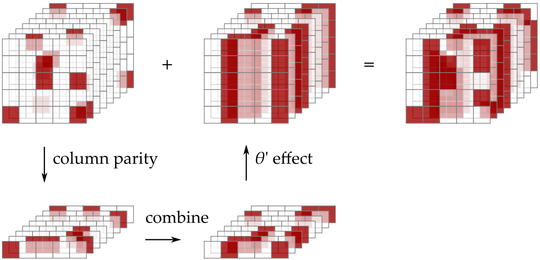
  - Global optimization saves more XORs
  - Old algorithms improve many results (e.g., AES MixColumns)
  - We find new MDS matrices with lowest number of XORs

Column Parity Mixers

- Keccak-f has very strong bounds on differential trails due to θ



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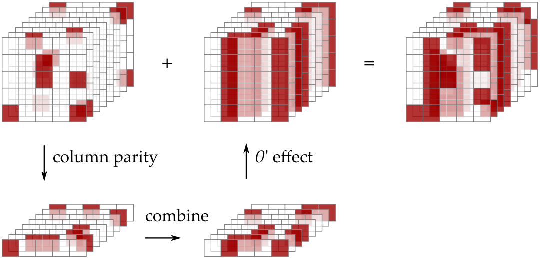

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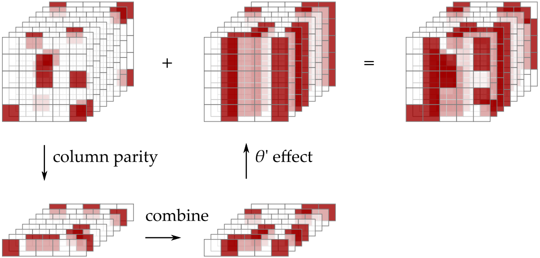

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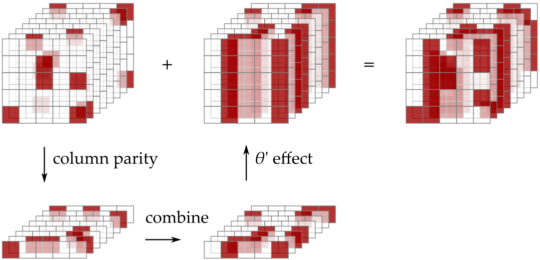

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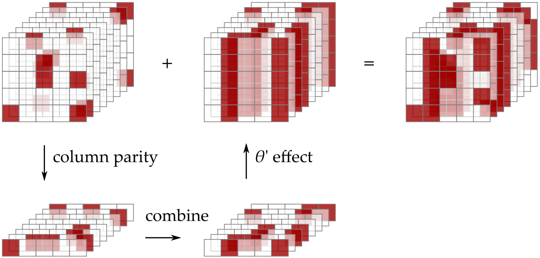

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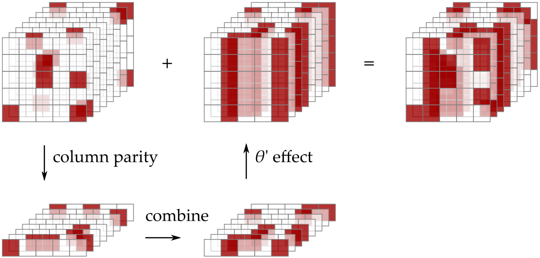

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- CPM: generalization of θ
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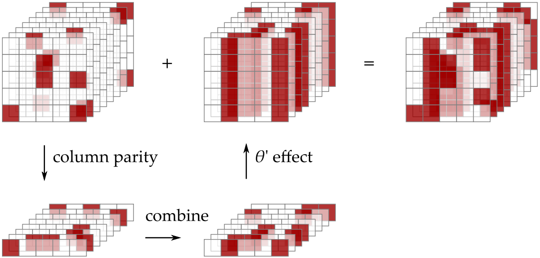
  - Good *diffusion* properties
  - Also suitable for strongly aligned ciphers
  - Competitive with MDS matrices



Column Parity Mixers

For an $m \times n$ matrix A over $\mathbb{F}_2^k$:

$$\theta(A) = A + \quad f(A)$$

$$\begin{pmatrix} a_{0,0} & a_{0,1} & a_{0,2} & a_{0,3} \\ a_{1,0} & a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,0} & a_{2,1} & a_{2,2} & a_{2,3} \end{pmatrix}$$

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θ fully defined by m, n and Z